Journal of Economic Behavior and Organization 10 (1988) 29–55. North-Holland

THE NECESSITY OF IMPERFECT DECISIONS

Ronald A. HEINER*

Brigham Young University, Provo, UT 84602, USA

Received March 1986, final version received October 1987

I construct a general model which neither postulates decisions are always optimal, nor that decision errors necessarily arise when 'real world' agents are involved. Nevertheless, I show that agents always have a positive marginal incentive to use some information imperfectly, but never to use all potential information even if they have costless access to perfect information about how to select every action. These results imply that in order for a decision problem to be internally consistent without simply postulating the extreme limit of perfect decisions, it must explicitly incorporate the effects of both information *and* decision errors on behavior.

1. Introduction

Conventional choice theory assumes agents respond to information perfectly in the sense of always making decisions that maximize expected utility based on their observed information. Opposing this view has been the persistent criticism that 'real world' agents have severe limitations in their ability to process information, which thereby prevents them from perfectly using information without error. This criticism is sometimes avoided by assuming decisions are adjusted to incorporate various 'transaction costs' of observing and processing information. Such cost-adjusted decisions can thereby still be regarded as optimal.[1]

Thus, the tendency has been to assert two opposite views: one which redefines optimizing so as to guarantee it will be satisfied; and one that regards imperfect decisions as the only plausible case for agents in the real world. As such, neither view allows the possibility of using information imperfectly itself to be analyzed.

Suppose, then, we investigate the case where agents can optimally use some but not necessarily all information potentially relevant to their decision problems. In this more general setting, *I show there always exists a positive*

*I would like to thank an anonymous referee for helpful and detailed comments on an earlier draft of this paper. Of course, I alone am responsible for the paper's content.

[1] However, there is a basic problem with assuming such decision-cost adjusted decisions are necessarily optimal; see footnote 7.

marginal incentive to use at least some information agents cannot respond to optimally. Consequently, the question of whether imperfect decisions applies in particular situations is not a matter of modeling discretion or convenience. Rather, it is one of generic necessity once the possibility of decision errors is permitted into the analysis; that is, once such errors are not excluded by hypothesis. This conclusion is developed in four steps.

Section 2 shows how to theoretically distinguish imperfect information from using it imperfectly, and develops a two-stage 'reliability' ratio which allows the interaction between these two sources of error to be explicitly modeled. Section 3 shows how this gives rise to a trade-off whereby more information better predicts the consequences of agents' decisions, but beyond a certain point using better information also produces increasing numbers of decision errors. Section 4 then presents a general result showing there always exists positive marginal benefits from using information beyond the threshold where decision errors begin (into the 'Imperfect Decision zone' or ID-zone).

On the other hand, agents are still guaranteed *not* to benefit from using all information even though it may be *costlessly* observable and in the limit perfectly predicts the consequences of their decisions. Imperfect agents can thus always benefit from going partially but never fully into the ID-zone. This result is also robust to introducing decision costs of reducing the incidence of decision errors. Such costs might result from enlarging memory, better sensory discrimination, increased computation speed, and so on. *So long as these costs are not increased to the limit where decision errors are completely eliminated* (so that the ID-zone vanishes) *the above results still hold.*

Given these results, section 5 asks how far different kinds of agents can potentially benefit from proceeding further into the ID-zone. As agents become more competent at using information they may benefit from proceeding *further* into the ID-zone. Conversely, less competent agents may only benefit from entering *less* into the ID-zone; that is, from restricting their use of information closer to the threshold where decision errors begin. Thus, relatively more competent agents may benefit from using larger amounts of more sophisticated information, but they may also use more than a negligible fraction of such information imperfectly. This means that more competent agents (such as humans compared to animals) are *not* necessarily those who will better approximate the behavior of optimal Bayesian decision makers.

2. Imperfect information versus using it imperfectly

The first objective is to distinguish imperfect information from *using it imperfectly*, where the former case has already been thoroughly analyzed in standard choice theory. To do so let the sets S, X, A denote respectively: possible *states* of the world, *information* about the true state of the world, and agents' repertoire of choosable *actions*. For each action $a \in A$, let S_a^*

denote those states for which action a is the best choice. That is, action a maximizes utility contingent on those states actually occurring. As a very simple example, suppose action a means bringing an umbrella. S_a^* would then comprise all those states of the weather where enough rain falls to make doing so preferred over not carrying an umbrella. Similar examples apply to any particular decision problem, such as when to adjust production or employment in response to underlying changes in relative prices, or when to enter into contractual commitments depending on a host of factors affecting the future outcome of such agreements.

Next, suppose agents must rely on information that may imperfectly signal which states will actually occur. Here also we can define X_a^* as the set of messages for which action a is the best choice; meaning those messages for which action a maximizes expected utility knowing only the 'posterior' probabilities of different states arising contingent on receiving those messages. Thus for example, depending on how barometer readings are imperfectly correlated with actual precipitation, there exists a range of pressure readings for which bringing an umbrella is preferred ex ante to not doing so, even though in particular instances it may or may not rain ex post. Similarly, observed prices may be a mixture of nominal price changes due to purely monetary disturbances, and 'genuine' relative price changes due to underlying 'real' factors. Depending on how observed prices noisily reveal true relative prices, there exist observed price changes for which adjusting production or consumption decisions is preferred to not doing so.

Finally, let the correspondence $B(x){:}X \rightarrow A$ represent a decision rule for choosing actions in response to observed messages; meaning an agent's *behavior* in responding to information. The usual practice is to postulate an optimal decision rule, denoted $B^*(x)$, which always maximizes expected utility contingent on received information. That is, $a \in B^*(x)$ if and only if $x \in X_a^*$ for all $a \in A$.

With the above notation we can introduce certain *reliability* concepts. First, consider the information potentially used by agents. Its reliability refers to how well the optimal messages for selecting an action distinguish between optimal and non-optimal states for selecting that action. This is determined by the following conditional message probabilities: $r_a^X = p(X_a^* \mid S_a^*)$ and $w_a^X = p(X_a^* \mid S - S_a^*)$. r_a^X is the chance of optimal messages being observed when optimal states for selecting action a occur. Similarly, w_a^X is the chance of optimal messages being observed when non-optimal states for selecting action a occur. The ratio $\rho_a^X = r_a^X / w_a^X$ thus measures the ability of messages X_a^* to correctly reveal the optimal states for choosing action a without mistakenly arising under non-optimal states for choosing it.[2] Perfect

[2]Using X to superscript r_a^X, w_a^X does *not* mean that the set of messages X is conditional on deciding to select any particular action $a \in A$ (nor does X necessarily depend on the decision rule $B(x)$ used by an agent). Regardless of whether X is determined prior or concurrently with deciding how to react to particular messages $x \in X$ (or determining what decision rule $B(x)$ to

information means $r_a^X = 1$ and $w_a^X = 0$ for all a; so that $\rho_a^X = r_a^X / w_a^X = \infty$ for all a.

Now apply the concept of reliability directly to agents' behavior *in responding to information.* Namely, how likely are agents to choose actions when optimal messages for doing so are observed without mistakenly selecting them when non-optimal messages for doing so arise? This will in general depend on the type of decision rule $B(x)$ that governs agents' behavior in responding to information. Thus, define the following conditional response probabilities, where their dependence on $B(x)$ is notationally indicated by *super*scripting in the following manner: $r_a^B = p(a \in B(x) \mid X_a^*)$, $w_a^B = p(a \in B(x) \mid X - X_a^*)$, and $\rho_a^B = r_a^B / w_a^B$. The ratio ρ_a^B measures the reliability of behavior at responding to the 'right' instead of the 'wrong' messages for choosing an action a, analogous to how ρ_a^X measures the reliability of information at signaling the right instead of the wrong states for choosing that action.[3] The limiting case of fully optimal decisions $B^*(x)$ corresponds

apply to potentially observed messages), the likelihood of rightly or wrongly selecting individual actions may depend on the type of information used by agents (as discussed in section 3). Hence, the explicit dependence indicated notationally by r_a^X, w_a^X and ρ_a^X for each $a \in A$.

[3]The explicit separation between information and decision reliability (ρ_a^X and ρ_a^B for $a \in A$) is not present in standard decision theory. Yet it is a fruitful distinction nevertheless. With it, we can distinguish between two basic sources of imperfection: from 'without' (or 'external errors') due to imperfect information embodied in messages received by agents; and from 'within' (or 'internal errors') due to internal limitations in agents' ability to assimilate and react appropriately to incoming messages.

The 'external' versus 'internal' distinction can be interpreted in various ways, two of which are especially relevant to standard decision theory. One is to view the information probabilities r_a^X and w_a^X as purely subjective to an agent (as 'subjective Bayesian probabilities'), embodying only personal beliefs about the reliability of messages in guiding when to select different actions $a \in A$. The possibility of $r_a^B < 1$ and $w_a^B > 0$ would then mean an internal inconsistency' in properly behaving in accordance with one's subjective beliefs about information (i.e., an agent's internal inference forming and reaction mechanisms are imperfectly coordinated with other observation and belief forming mechanisms). Alternatively, r_a^X and w_a^X could refer to objective statistical properties produced through external environmental relationships; akin to recent 'rational expectations' models. The probabilities r_a^B and w_a^B could then incorporate the effects of decision errors about how to correctly decide relative to objectively known statistical error rates of received messages. Or we might mix objective and subjective error interpretations by having r_a^X, w_a^X refer to objective statistical properties, while $r_a^B < 1$, $w_a^B > 0$ arises from agents forming mistaken subjective beliefs about r_a^X, w_a^X.

Still another possibility is that r_a^B and w_a^B measure an agent's *self*-awareness of its own imperfection in responding to information; where $r_a^B \equiv 1$ and $w_a^B \equiv 0$ for all a means an agent is perfectly self-confident at making all potential decisions. Imperfect self-confidence (measured by $r_a^B < 1$ and $w_a^B > 0$ for different actions) can then be explicitly incorporated into the analysis (thereby establishing a theoretical link to key areas of cognative psychology such as personality conflict, self-disception, depression, motivation therapy, and so on). Self-confidence may also affect an agent's willingness to take risks (that is, it may affect the degree of 'risk aversion' versus 'risk loving') with consequent effects on investment and entrepreneurial behavior. *Whatever may be the appropriate interpretation in different applications, the analytical objective is to model the behavioral consequences arising from both external and internal sources of error (rather than assuming agents behave as if only the former errors exist).* Hence, the motivation for an explicit analytical distinction between r_a^X, w_a^X and r_a^B, w_a^B for potentially chosen actions $a \in A$.

to $r_a^{B*}=1$ and $w_a^{B*}=0$, which implies $x \in X_a^*$ if and only if $x \in B^*(a)$.[4] In this case agents are said to be 'perfectly reliable' at using information; so that $\rho_a^{B*}=\infty$ for all $a \in A$.

Next, consider the joint interaction between imperfect information ($\rho_a^X < \infty$) and using it imperfectly ($\rho_a^B < \infty$). Let $r_a^{XB}=p(a \in B(x) \mid S_a^*)$ denote the likelihood of selecting an action a when optimal states for doing so actually arise. Note that this implicitly depends on how agents imperfectly respond to information and on how information is imperfectly correlated with particular states. Similarly, let $w_a^{XB}=(a \in B(x) \mid S-S_a^*)$ denote the likelihood of choosing an action a when non-optimal states for doing so actually arise. The ratio $p_a^{XB}=r_a^{XB}/w_a^{XB}$ then measures the relative likelihood of selecting an action under optimal instead of non-optimal states for doing so, as *jointly* affected by imperfect information *and* using it imperfectly. It can be decomposed into the following general formula (see part D of the appendix):

Theorem 1. (*The Structure of Joint Reliability*)

$$\rho_a^{XB}=\frac{r_a^X(\rho_a^B-1)+1}{w_a^X(\rho_a^B-1)+1} \quad \textit{for all} \quad a \in A. \tag{1}$$

Formula (1) implies a direct trade-off between the reliability of information and agents' reliability at using it: less reliable use of information will reduce their joint reliability at choosing an action in response to any potential information source. To see this, note that as $\rho_a^B \to 1$, both the numerator and denominator of ρ_a^{XB} necessarily also approach 1 regardless of how close w_a^X and r_a^X might approach 0 and 1 respectively; that is, regardless of how large ρ_a^X might be. In addition, as long as messages have at least a 50–50 chance of correctly signaling when to select an action, so that $\rho_a^X \geq 1$, then joint reliability ρ_a^{XB} necessarily falls *below* ρ_a^X as ρ_a^B drops below infinity.[5] Thus, *imperfectly using information necessarily reduces agents' joint reliability below that of any partially informative information source, eventually to the point where they have only a 50–50 chance of choosing actions when optimal instead of nonoptimal to do so. The latter result holds no matter how reliable information might be on its own.*

[4] *If* $x \in X_a^*$ and $r_a^B=\rho(a \in B(x) \mid X_a^*)=1$, then $a \in B(a)$ must hold.
Conversely, if $x \in X-X_a^*$ and $w_a^B=\rho(a \in B(x) \mid X-X_a^*)=0$, then $x \notin B(x)$ must hold. Hence, $r_a^B=1$ and $w_a^B=0$ imply $x \in X_a^*$ if and only if $a \in B(x)$ (or equivalently, $a \in B(x)$ if and only if $a \in B^*(x)$; since $a \in B^*(x)$ by definition if and only if $x \in X_a^*$).

[5] Substitute formula (1) into the inequality $\rho_a^{XB} \leq \rho_a^X = r_a^X/w_a^X$, and cross multiply the numerators and denominators assuming that ρ_a^B is *finite*. After cancelling common terms and rearranging the inequality reduces to simply $1 \leq r_a^X/w_a^X=\rho_a^X$. Thus, if $\rho_a^B<\infty$, then $1 \leq \rho_a^X$ is the only requirement needed for the original inequality $\rho_a^{XB} \leq \rho_a^X$ to hold.

Besides the above properties, another reason for analyzing joint reliability ratios ρ_a^{XB} is their relationship to the expected utility achieved from selecting different actions in response to observed information.[6] Let $EU(A|X)$ denote the expected utility from selecting actions $a \in A$ in response to observed messages $x \in X$. If joint reliability is rising *simultaneously* for all actions, meaning ρ_a^{XB} is increasing for all $a \in A$, then $EU(A|X)$ will also rise (see the part H of the Appendix). For example, suppose messages more reliably indicate when to select every action, so that ρ_a^X increases for all $a \in A$. In addition, suppose agents decide at least as reliably as before, so that ρ_a^B does not fall for any $a \in A$. Then the expected utility over all decisions based on such improved information $EU(A|X)$ will necessarily also rise.

Note, however, that rising $EU(A|X)$ does not by itself incorporate any potential costs of acquiring or observing better information. Consequently, $EU(A|X)$ measures only the expected gain or 'marginal benefit' from using better information. As discussed below, the objective is to determine when the marginal benefit from using better information is positive or negative. Doing so will thereby determine whether there is a positive or negative incentive to use or search for more information. This type of result can be obtained without trying to formulate a second or third stage 'meta-optimization' problem where agents try to jointly determine how to react to observed information, along with determining the optimal amount of information to observe and the optimal level of search and observation costs.

The reason for not formulating such a multistage optimization problem is that doing so introduces additional search and observation dimensions whose associated costs and benefits further expand the number of optimizing margins beyond those already present in trying to respond optimally to any given information set. Thus, adding additional search and information cost margins even further increases the complexity of the resulting multi-level decision problem compared to ignoring them. Consequently, *the frequency of decision errors may even further increase if imperfect agents try to solve such a multi-level optimization problem.*[7]

[6] See footnote 11.

[7] This is an instance of a general problem whereby adding additional levels of costs and benefits of calculating previously included optimizing margins produces an infinite regress to higher 'meta' decision levels. The reason for such an infinite regress is that comparing costs versus benefits of eliminating decision errors for each next decision level *further expands the number of optimizing margins agents must attempt to keep track of compared to those present before decision errors were considered in the first place.* Thus, the complexity of agents' expanded decision problem increases relative to whatever decision skills were originally at their command before additional optimal decision-error margins were introduced. Consequently, additional decision errors will arise with each new decision error optimizing margin, instead of eventually converging to zero when enough such cost-benefit margins are introduced (that is, *decision costs bring with them more optimizing margins and thus more potential decision errors in trying to control them*).

The latter conclusion means that no matter how many types of decision processing or information costs are introduced (with their associated optimizing margins) still further decision

We can avoid these problems by focusing instead on a more basic question; namely, *when will positive marginal benefits to using more information exist*, thereby creating a positive incentive for responding to more information? This question can be analyzed without assuming imperfect agents somehow behave as if they solved an even more complicated multi-level decision problem than that which they were already unable to solve before introducing further optimizing margins in the first place.

3. Better information versus more decision errors: A basic trade-off

As discussed in the introduction, suppose agents can perfectly use some but not all potentially available information. Before proceeding to the main results, three interpretations of this general possibility are briefly described and then formalized with the reliability concepts introduced above.

3.1. Finite channel capacity

One of the basic results of information and cybernetic theory concerns transmitting information through an imperfect channel which tends to garble information. It is possible to encode any set of messages so as to reduce transmission errors arbitrarily close to zero up to the channel's capacity to transmit information. Beyond this limit, errors will necessarily rise no matter how messages are transmitted [see Theorem 11 of Shannon and Weaver (1963)]. Suppose agents' decision processes represent an information channel of finite capacity which attemps to transform messsages into optimally selected actions. That is, agents use messages as 'inputs' to generate 'outputs' in the form of actions which are perfectly correlated with the optimal messages X_a^* for selecting them; so that $r_a^B=1$, $w_a^B=0$ and $\rho_a^B=\infty$ for all $a\in A$ (see parts E and F of the appendix for a brief formal statement). The above theorem means that agents may be able to perfectly use information up to a certain amount of messages received as input. However, beyond this point, responding to still further messages will produce decision errors which reduce r_a^B below one and raise w_a^B above zero. This in turn implies that the reliability ratios ρ_a^B for selecting different actions now drop below their upper limit of infinity and continue to fall as more information is used.

3.2. Information complexity

Besides the amount of information agents use as input, they may also have limited skills at interpreting or discriminating between certain kinds of

errors will remain whose behavioral consequences can be explicitly studied. Accordingly, this paper focuses on a key aspect of studying *imperfect* choice, rather than assuming agents behave optimally relative to some higher level decision problem. Further discussion of these issues is contained in Heiner (1983, pp. 569–70; 1985, pp. 91–92; 1987c, pp. 6–7, 16–17).

messages. In particular, what kind of information is needed to predict a subtle, continually changing environment? It cannot be tracked by a simple binary message such as on–off, black–white, and so on. More complex compound messages can by built up from simpler messages, but then agents may have to distinguish between a very large number of possible compound signals that can arise through different, possibly subtly differentiated, patterns of simpler components. Thus, in order for messages to better predict environmental changes, they may themselves become more complex and thereby more difficult for agents to interpret correctly (for a formal development using concepts from information theory, see part G of the appendix). Consequently, as messages become sufficiently complex, agents can no longer interpret them without error. In terms of the reliability ratios ρ_a^X and ρ_a^B, this means that in order for information to become more reliable its own complexity may increase and thereby at some point reduce agents' reliability at using it. Consequently, as the information reliability ratios ρ_a^X rise above some threshold, the corresponding decision reliability ratios ρ_a^B drop below infinity and continue to fall thereafter.

3.3. Nonlocal information

Suppose agents have some ability to learn from their prior experience in responding to information. For example, their reliability at using messages improves through repeated use and exposure to them. Agent's past experience will then have a biasing effect on their ability to use information even when processing a fixed amount of information, or the messages involved are of equal complexity. This is a special case of a general principle whereby agents' reliability at using information, measured by the ratios ρ_a^B for $a \in A$, will at some point drop as it becomes sufficiently 'non-local' in some dimension from the recurrent features of their ongoing experience[8] [see Heiner (1985c, also 1986b, 1987a)]. For example, individual agents may have frequent dealings in only a few markets within a larger economic system. Consequently, as various transaction messages (such as market prices and quantities) expand beyond these 'familiar' markets, agents may at some point become less and less reliable at using them. On the other hand, such an expanding set of transaction messages X, *if correctly interpreted*, will better predict market conditions over the whole economy. Thus, at some point

[8]Differential sensitivity to information depending on prior exposure or similarity to other familiar messages is the focus of several literatures in experimental psychology and animal behavior. See for example the studies of 'exposure effects' in R. Zajonc (1968, 1980), and J. Seamon, N. Brody and D. Kauff (1983); the studies of 'perceptual set and expectancy effects' in U. Neisser (1976); and studies of 'search images' and 'generalization gradients' by D. McFarland (1985) and N.J. Mackintosh (1974). Closely related to these studies is the work of Richard Day on adaptive dynamic search behavior; such as Day (1984) and the references cited therein. Local dynamic search also plays a key role in the 'satisficing' theories of Herbert Simon (1957, 1983), and in Richard Nelson and Sidney Winters 1982 book on evolutionary economic change.

expanding X will both further raise ρ_a^X for particular actions, while also causing ρ_a^B to drop below infinity and continue falling.[9]

Let us now formalize the above three examples. The variable z is used to index changes in the set of information X, or in the type of individual messages contained in X. Thus, larger values of z could measure: the amount of information agents process as input; the complexity of received messages; or the degree of familiarity or non-localness of these messages. Note that expanding X may involve changes in all three factors. That is, larger X may increase the number of messages for agents to process, which may themselves be more complex or less familiar than previously included messages. The conditional probabilities r_a^X, w_a^X, r_a^B, w_a^B for $a \in A$ are assumed differentiable functions of z. Derivatives with respect to z are denoted with a dot above the corresponding variable, and z ranges over the interval $[\bar{z}, \infty]$.

The probabilities r_a^X, w_a^X start off initially *equal* at $\bar{z}$. This corresponds to uniformative messages which arise with equal chance whether optimal states for selecting an action arise or not (so that $\rho_a^X(\bar{z}) = 1$). The reliability of information also rises monotonically with z toward infinity. That is, the derivatives of r_a^X and w_a^X satisfy $\dot{r}_a^X > 0$, $\dot{w}_a^X < 0$ for all $z \geqq \bar{z}$, and $r_a^X \to 1$, $w_a^X \to 0$ as $z \to \infty$. This implies $\dot{\rho}_a^X > 0$ for all $z \geqq \bar{z}$, and $\rho_a^X \to \infty$ as $z \to \infty$. Thus, for each action $a \in A$, ρ_a^X starts off at one and rises strictly toward infinity as z increases. On the other hand, agents start off perfectly using information as z increases up to an 'error threshold', denoted z^0, where decision errors begin to occur and thereafter accumulate for larger values of z. That is, $r_a^B(z) = 1$, $w_a^B(z) = 0$ for all actions $a \in A$ and $\bar{z} \leqq z \leqq z^0$.

However, beyond z^0 there is at least one action, denoted a^0, for which $\dot{r}_{a^0}^B(z) < 0$ and $\dot{w}_{a^0}^B(z) > 0$ for all $z > z^0$. If this happens at z^0 for more than one action simultaneously, then action a^0 can be arbitrarily selected from among this group. As z rises further beyond z^0, decision errors will begin accumulating for successively more actions, until eventually $\dot{r}_a^B < 0$ and $\dot{w}_a^B > 0$ for all $a \in A$. This implies agents' reliability at using information ρ_a^B remains infinite for all actions in A until z reaches z^0. Thereafter, the decision reliability ratios for different actions successively drop below infinity, beginning with $\rho_{a^0}^B$ which starts dropping at z^0.

Next consider the information reliability ratio for action a^0, $\rho_{a^0}^X$. Since $\rho_{a^0}^X$ rises monotonically with z, we can also define the error threshold as that point where $\rho_{a^0}^X(z)$ rises to $\rho_{a^0}^X(z^0)$; denoted $\rho^0 = \rho_{a^0}^X(z^0)$. In the discussion that follows it will be convenient to divide the range of $\rho_{a^0}^X(z)$ into two intervals: the closed interval $[1, \rho^0]$ where no decision errors occur as information becomes more reliable; and the open interval (ρ^0, ∞) where decision errors accumulate for different actions as $\rho_{a^0}^X$ rises further beyond ρ^0.

The general picture is thus one of increasingly reliable information which

[9]For an application of these concepts to rational expectation models of the business cycle, see Heiner (1986a).

at some point becomes too difficult for agents to use without error, thereafter steadily lowering their reliability at using it to select particular actions $a \in A$. The initial phase before decision errors begin is called the *Perfect Decision zone*, or PD-zone. The second phase where decision errors accumulate is called the *Imperfect Decision zone*, or ID-zone.

4. The necessity of imperfect decisions

As discussed in the introduction, we could eliminate the ID-zone by assuming it doesn't exist; that is, by postulating agents always optimally use any amount or type of information. Alternatively, we could assume it is prohibitively costly to observe any information in the ID-zone. Thus, in order to analyze the possibility of using imperfect information, assume agents have access to a range of costlessly observable information sources for which an ID-zone exists as described above.

Recall from section 2 that agents' joint reliability ρ_{a0}^{XB} equals the reliability of information ρ_{a0}^{X} if they perfectly use it, but falls below ρ_{a0}^{X} if any decision errors occur. That is $\rho_{a0}^{XB}=\rho_{a0}^{X}$ when $\rho_{a0}^{B}=\infty$, but $\rho_{a0}^{XB}<\rho_{a0}^{X}$ when $\rho_{a0}^{B}<\infty$, dropping to one as ρ_{a0}^{B} drops to one. Fig. 1 depicts these relationships by graphing *both* $\rho_{a0}^{XB}(z)$ and $\rho_{a0}^{X}(z)$ as z varies above its lower limit $\bar{z}$. This produces a path which simultaneously shows how ρ_{a0}^{XB} and ρ_{a0}^{X} vary relative to each other as ρ_{a0}^{X} ranges over the lower axis between one and infinity. Up to the error threshold ρ^{0}, ρ_{a0}^{XB} and ρ_{a0}^{X} equal each other and thus rise together along the 45° line. Thereafter ρ_{a0}^{XB} falls below ρ_{a0}^{X} for $\rho_{a0}^{X}>\rho^{0}$. The same qualitative relationship holds for any other action $a \in A$; except that the point where $\rho_{a}^{XB}(z)$ falls below $\rho_{a}^{X}(z)$ may occur at values of $\rho_{a0}^{X}(z)$ that exceed $\rho^{0}=\rho_{a0}^{X}(z^{0})$.

The curve graphed in fig. 1 also has a unimodal or 'single peaked' shape. This property does not automatically hold without further assumptions which are developed in part C of the appendix. Briefly, sufficient conditions are: (1) that r_{a}^{X} and w_{a}^{X} shift respectively up and down toward 1 and 0 at *decelerating* rates, meaning $\ddot{r}_{a}^{X}<0$ and $\ddot{w}_{a}^{X}<0$ (where double dots $\ddot{s}$ mean second derivatives with respect to z); (2) the absolute percentage drop in w_{a}^{X} toward zero does not exceed the absolute percentage increase in r_{a}^{X} toward one, meaning $|\dot{w}_{a}^{X}/w_{a}^{X}|\leq|\dot{r}_{a}^{X}/r_{a}^{X}|$; and (3) the negative percentage change in the derivative of decision reliability $\dot{\rho}_{a}^{B}$ does not exceed twice the absolute percentage change in $(\rho_{a}^{B}-1)$, meaning $|\ddot{\rho}_{a}^{B}/\dot{\rho}_{a}^{B}|\leq|2\dot{\rho}_{a}^{B}/(\rho_{a}^{B}-1)|$.

Now consider how much information imperfect agents potentially may benefit from using. In particular, will they benefit from using information beyond the threshold where decision errors begin to occur? I will not answer this question by formulating a multi-level decision problem simultaneously determining different information sets X, and their associated observation costs along with the decision rule $B(x)$ for reacting to messages in X. As

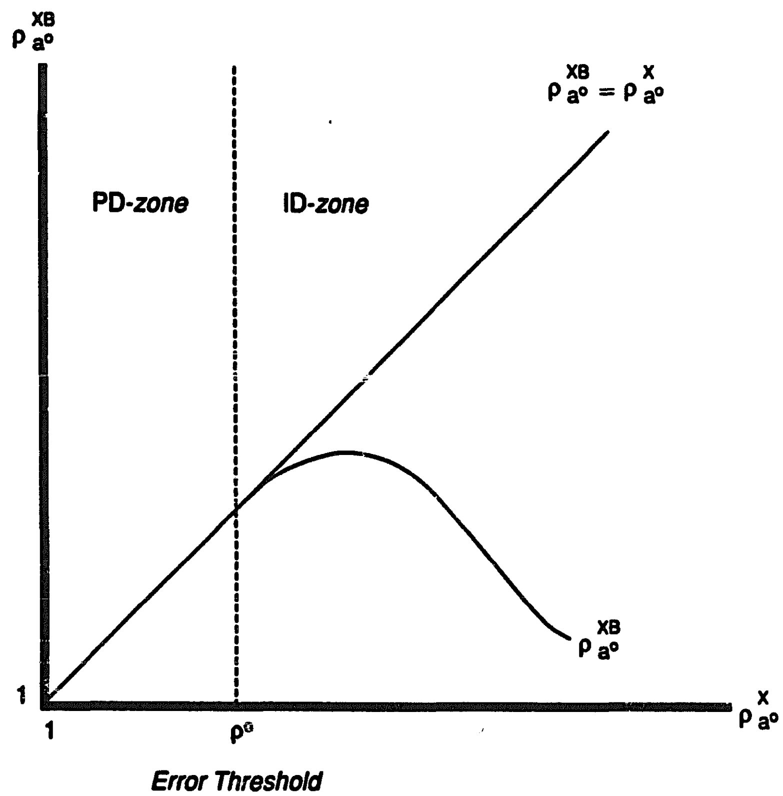

Fig. 1. Agents' joint reliability $\rho_{a^0}^{XB}$ initially equals the reliability of information $\rho_{a^0}^{X}$ because they optimally respond to messages (that is, $\rho_{a^0}^{B}=\infty$) up to the error threshold ρ^0. Beyond ρ^0 they begin to make decision errors (so that $\rho_{a^0}^{B}$ is now finite and falling), which causes $\rho_{a^0}^{XB}$ to steadily drop below $\rho_{a^0}^{X}$; eventually not only relative to $\rho_{a^0}^{X}$ but also absolutely down toward one. As shown, $\rho_{a^0}^{XB}$ always reaches a maximum for some finite $\rho_{a^0}^{X}>\rho^0$.

already discussed in section 3, imperfect agents are even less able to solve such a multi-level problem than the initial problem of how to respond to any given information set. However, we can precisely analyze whether there exists a positive marginal benefit to using information beyond the decision error threshold represented by ρ^0 in fig. 1. Doing so will establish whether there exists a positive incentive for agents to use at least some information imperfectly. As will be shown, the answer to this question is necessarily *yes* (under certain regularity conditions about decision errors 'smoothly' beginning).

One reason for posing this question is a methodological one about the usual modeling assumptions of statistical decision theory. Namely, a generic 'yes' answer means *it is illegitimate to assume statistical decisions are always optimal* (corresponding to $\rho_a^B=\infty$ for all $a\in A$) *once the possibility of decision errors is not excluded by hypothesis, even assuming agents have costless access to a range of information sources which they can use perfectly.* Despite the option of using only the latter information, imperfect agents will always have a positive marginal incentive to use information beyond the point where decision errors never occur; that is, *beyond the point where the usual assumption of optimal decisions is satisfied.* Consequently, we now have a basic theoretical reason for explicitly analyzing the behavioral effects of imperfect decisions, rather than assuming agents always respond to information perfectly.

Such a theoretical justification further supports a general theme I have elsewhere introduced [Heiner (1983, 1985c, 1986b, 1987b)] about predictable behavior arising from imperfect choice. The reason is that *decision errors create potential benefits from controlling them successfully.* Such errors thereby produce systematic incentives toward controlling decisions with rules and procedures that discipline behavior into relatively more predictable patterns than would otherwise result if there were no decision errors to potentially regulate in the first place. Consequently, *analyzing the effects of decision errors becomes a powerful new explanatory source for predicting behavior; one that cannot be used so long as we continue to postulate that decisions are always optimal.* Hence, the motivation for showing in this paper a further theoretical justification for analyzing imperfect decisions.

In order to intuitively understand this justification, consider the qualitative relationship between $\rho_{a^0}^{XB}$ and $\rho_{a^0}^{X}$ shown in fig. 1. Note that up to the error threshold ρ^0, $\rho_{a^0}^{XB}$ rises *one-to-one* with $\rho_{a^0}^{X}$; where the same relationship holds for any other $a\in A$, since $\rho_a^B=\infty$ for all $a\in A$ up to ρ^0. Thus, at the error threshold ρ^0, either the joint reliability ratios ρ_a^{XB} will continue to rise one-to-one if $\rho_a^B=\infty$ continues to hold, or they will begin dropping below their respective information reliability ratios ρ_a^X if their decision reliability ratios ρ_a^B drop below infinity beyond ρ^0. $\rho_{a^0}^{XB}$ is either the first or among the first joint ratios for which this happens.

These relationships imply that at ρ^0, *the joint reliability of selecting all actions will continue to rise unless one or more of the latter ratios such as $\rho_{a^0}^{XB}$ immediately switches from rising one-to-one to strictly falling just as the error threshold is reached.* This can happen only if the denominator w_a^B of one of the latter ratios has a *discontinuous* derivative that instantly jumps from zero to a strictly positive level. Hence, so long as decision errors 'smoothly' begin for all actions, there will exist a positive marginal incentive to using information beyond the error threshold ρ^0.

On the other hand, the previously discussed properties of the ρ_a^{XB} ratios also guarantee they will *all* eventually fall toward one as more information is used, since $\rho_a^{XB}\to 1$ as $\rho_a^B\to 1$ for all $a\in A$. Consequently, the marginal benefits from using more information to guide selections of different actions will eventually become negative for all $a\in A$. We therefore have the following general result (proven in part A of the appendix):

Theorem 2. (*The Necessity of Imperfect Decisions*)

(*a*) *Let a^0 be any action for which $\rho_{a^0}^B$ starts dropping below infinity at ρ^0,* *and let the derivatives $\mathring{r}^B_{a0}$, $\mathring{r}^X_{a0}$, $\mathring{w}^X_{a0}$ have finite right hand limits at ρ^0. Then the joint reliability ρ^{XB}_{a0} will immediately begin dropping at the error threshold ρ^0 only if $\mathring{w}^B_{a0}$ instantly jumps from zero to a strictly positive level given by the inequality,*

$$\mathring{w}^B_a > \frac{\mathring{r}^X_a w^X_a - \mathring{w}^X_a r^X_a}{r^X_a - w^X_a} > 0,$$

where $^\circ$ denotes the right hand derivative as z approaches z^0 from above (corresponding to ρ^X_{a0} approaching ρ^0 from above in fig. 1).

(b) The above inequality implies that if w^B_{a0} is continuously differentiable at ρ^0, so that $\mathring{w}^B_{a0}$ 'smoothly' rises from zero at ρ^0, then the joint reliability ratios of all *actions will continue rising beyond the error threshold ρ^0. Consequently, so long as decision errors 'smoothly' begin, there always exists a positive marginal benefit from using information strictly into the imperfect decision or ID-zone.*

(c) Moreover, the joint reliability ratios ρ^{XB}_a for all *actions in A will eventually reach finite maximums and thereafter drop monotonically to one. Consequently, the marginal benefit from using more information will always fall below zero* before *agents reach the limit of using perfectly reliable information. That is, positive marginal benefits from using more reliable information will continue at most* partially *into the ID-zone even though information eventually becomes perfectly reliable, and even if all potentially usable information is* costless *to observe.*

As discussed above, Theorem 2 implies that once we extend standard choice theory to allow the possibility of decision errors, agents will in general *not* limit their use of information so as to behave as previously assumed. It should also be emphasized that this basic conclusion still applies even if we allow agents the opportunity of using resources to improve their information processing skills; such as enlarging memory capacity, better sensory discrimination, faster and more accurate computations, and so on. In particular, let C_d be the sum of decision costs associated with such improved skills. C_d does *not* refer to costs of searching for more information, but rather to costs of correctly interpreting and reacting to information once it has been observed. Assume higher C_d has the mutual effect of delaying the error threshold ρ^0 and reducing the rate of decision errors beyond ρ^0. This will shift the relationship between ρ^{XB}_{a0} and ρ^X_{a0} shown in fig. 1; with similar shifts applying to other actions in A. Three such relationships for successively higher values of C_d are shown in fig. 2. Higher decision costs reduce the ID-zone by raising the error threshold ρ^0. But so long as it doesn't vanish, Theorem 2 still guarantees agents will have a positive marginal incentive to imperfectly use some but not all potential information.

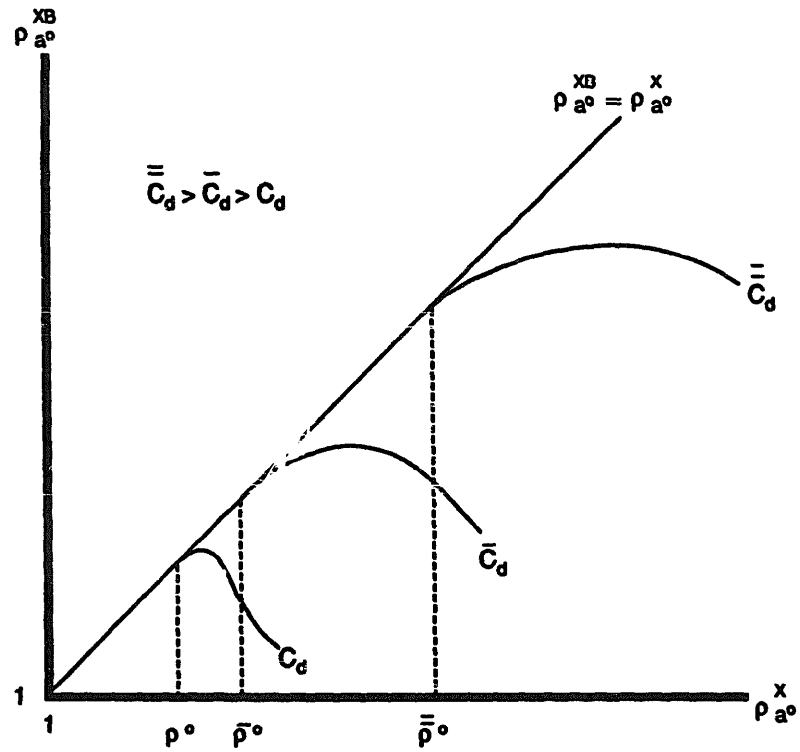

Fig. 2. Higher decision costs C_d improve agents' competence at using larger amounts of more complex and non-local information, thereby delaying the error threshold ρ^0 and slowing the drop in joint reliability $\rho^{XB}_{a^0}$ below $\rho^{X}_{a^0}$ after ρ^0 is reached. Three curves depicting this pattern are shown for successively higher $C_d < \bar{C}_d < \bar{\bar{C}}_d$. Note how each curve peaks further beyond its corresponding error threshold as C_d increases.

5. Agents of differing competence

The curves shown in fig. 2 can be interpreted not only as a *single* agent using resources to improve its decision skills, but also as *distinct* agents with different levels of decision costs embodied in more efficient decision mechanisms. This might involve 'hardwired' differences in neurological design such as between different biological species, or 'software' differences in message processing or interpretation methods learned through ongoing experience. Whatever the interpretation, fig. 2 illustrates a general principle: *more competent agents will have a positive marginal benefit from using information relatively further into the ID-zone than less competent agents.* The reason is that as agents become better at using information, they can respond to more complex and finely differentiated messages before mistakes in selecting particular actions start to occur, while also reducing the frequency of such errors as still further information is used. *That is, the threshold ρ^0 increases, and ρ^{XB}_a drops below ρ^X_a more slowly beyond ρ^0 than before, for any $a \in A$* (see part C of the appendix for a precise statement). On the other hand, as agents become less competent, decision errors begin sooner and more quickly accumulate as further information is used.

Think of the above also in terms of the common belief that agents with better decision skills will more closely approximate optimal Bayesian decision makers. This view is only partially correct. Better decision skills will delay the error threshold, but also enable agents to benefit from going further beyond it. Consequently, *more skillful agents may end up using more information, but a larger fraction of that information is used imperfectly compared to less competent agents who respond to less information, but also use relatively more of it without error.* Thus, at one extreme we may have highly sophisticated agents responding to a large number of complex messages, a substantial proportion of which are used imperfectly. On the other hand, a very simple agent might respond only to a relatively small number of crude but easily interpreted messages, virtually all of which are used perfectly.

The latter possibility fits a number of well-known cases of *releasing* behavior in animals. This usually consists of simple messages (involving color, shape, movement and so on) triggering certain responses independent of other much more informative, but also more complicated messages. For example, consider the following summary of Niho Tinbergen's studies of fighting behavior between male stickleback fish [Keeten (1976, page 504; for other examples see also pages 502–506, 496–498)].

> In the spring the throat and belly of the males become intensely red [suggesting] that the red color was an important stimulus. The investigators presented their subjects with a series of models, some quite like actual male sticklebacks except that they lacked the red coloration, and some showing little resemblance to actual sticklebacks except that they were red on the lower surface. The male fish attacked the red-bellied models, despite their unfishlike appearance, much more vigorously than they did the fishlike ones that lacked red. Surely the sticklebacks could see the other characteristics of the models, but they reacted essentially only to the releasing stimuli from the red belly.

The above example is only one out of a number of possibilities as we proceed across different species. In particular, the higher primates (especially humans) will typically not benefit from being so severely restricted in using information. Instead, they may benefit from using not only richer, more sophisticated messages, but also from information 'sensitivity' possibly well beyond the error threshold which defines the boundary of the ID-zone. This may in fact be the generic case when humans are involved.

6. Conclusion

I have constructed a model which neither postulates decisions are always optimal, nor that decision errors are the only plausible case when 'real world' agents are involved. Instead, agents have access to a set of available information, only part of which they can respond to perfectly. As the amount of information they use to guide their decisions increases sufficiently, or the messages involved become sufficiently complex or nonlocal to their ongoing experience, eventually a threshold is reached where decision errors begin and further accumulate thereafter. Information beyond this threshold is thus called the *Imperfect Decision* or ID-zone.

So long as the ID-zone exists and decision errors smoothly begin at its threshold, imperfect agents will always have a positive marginal benefit from continuing partially but never completely into the ID-zone. The latter result implies imperfect agents with *not* benefit from using all information. Moreover, this conclusion holds even if the ID-zone in the limit contains perfect information about how to select every action, and even if all potentially available information is *costless* to observe. The generic case for imperfect agents is thus to use information somewhere into the *interior* of the ID-zone. In addition, as agents become more competent at using information, they may benefit from proceeding relatively *further* beyond the error threshold.

The above results provide a basic theoretical justification for investigating the behavioral effects of imperfect decisions. Namely, *once the possibility of decision errors is not excluded by hypothesis, there always exists a positive marginal incentive to using information beyond the point where it is legitimate to assume optimal decisions.* Thus, in order for a decision problem to be internally consistent without simply postulating the extreme limit of perfect decisions, it must explicitly incorporate the effects of both information *and* decision errors on behavior.

Appendix

A. *Proof of Theorem 2*

Start from a value of z greater than z^0 so that ρ^B_{a0} is finite, and differentiate the formula for ρ^{XB}_{a0} given in eq. (1). To simplify calculations, let $r^X_{a0}=A$, $w^X_{a0}=B$, $r^B_{a0}=r$, $w^B_{a0}=w$, $\rho=r/w$, $r^{XB}_{a0}/w^{XB}_{a0}=\gamma$.

These definitions imply $\gamma=(A(\rho-1)+1)/(B(\rho-1)+1)$, from which we then have,

$$\dot{\gamma}\leqq 0\Rightarrow[B(\rho-1)+1][A\dot{\rho}+\dot{A}(\rho-1)]-[A(\rho-1)+1][B\dot{\rho}+\dot{B}(\rho-1)]\leqq 0$$

$$\Rightarrow AB(\rho-1)\dot{\rho}+\dot{A}B(\rho-1)^2+A\dot{\rho}+\dot{A}(\rho-1)-AB(\rho-1)\dot{\rho}-A\dot{B}(\rho-1)^2$$

$$-B\dot{\rho}-\dot{B}(\rho-1)\leqq 0$$

$$\Rightarrow (\rho-1)^2(\dot{A}B-A\dot{B})+\dot{\rho}(A-B)+(\rho-1)(\dot{A}-\dot{B})\leqq 0$$

$$\Rightarrow -\dot{\rho}(A-B)\geqq(\rho-1)^2(\dot{A}B-A\dot{B})+(\rho-1)(\dot{A}-\dot{B}) \tag{A.1}$$

$$\Rightarrow \frac{-\dot{\rho}}{(\rho-1)^2}\geqq\frac{\dot{A}B-A\dot{B}+(\dot{A}-\dot{B})/(\rho-1)}{A-B}. \tag{A.2}$$

In addition, the left-hand side of (A.2) expands as follows,

$$\frac{-\dot{\rho}}{(\rho-1)^2}=\frac{-(w\dot{r}-r\dot{w})}{w^2}\bigg/\left(\frac{r}{w}-1\right)^2=\frac{-w\dot{r}+r\dot{w}}{(r-w)^2}. \tag{A.3}$$

Then take the limit of (A.2) and (A.3) as z approaches z^0 from above (which corresponds to the right-hand derivative denoted with a ^{0} in Theorem 2). Recall that doing so implies $r\to 1$, $w\to 0$, and $\rho=r/w\to\infty$. Recall also that A and B are probabilities bounded between one and zero, and that the right hand limits of their derivatives $\dot{A}$ and $\dot{B}$ converge to finite limits, denoted $\mathring{A}$ and $\mathring{B}$. Consequently, as $z\to z^0$ the right-hand side of (A.2) reduces to $(\mathring{A}B-A\mathring{B})/(A-B)$. This expression is positive since $\dot{A}>0$ and $\dot{B}<0$ for all $z\geqq\bar{z}$ by hypothesis, which in turn implies $A-B>0$ at z^0 since $A=B$ at $\bar{z}<z^0$. Since the right hand limit of $\dot{r}$ is also finite (and since r and w also approach 1 and zero respectively), then (A.3) reduces to the right hand limit of $\dot{w}$, denoted $\mathring{w}$, as $z\to z^0$. Thus, substituting the last two results into (A.2) establishes part (a) of the theorem.

Part (a) implies that for any action a such that $\mathring{w}_a>0$ for $z>\hat{z}$, its joint reliability ρ_a^{XB} will still strictly rise at z^0 if $\dot{w}_a$ is continuous at z^0 (so that the inequality derived for part (a) *fails* to hold). Since the joint reliability ratios of other actions are still rising one-to-one with ρ_a^X, then part (a) implies the joint ratios of *all* actions continue to rise beyond the error threshold z^0 (or $\rho^0=\rho_{a^0}^X(z^0)$ in fig. 1). Hence, the expected utility from selecting actions in response to observed messages must still be rising as z increases beyond z^0 (i.e., there is still a positive marginal benefit from using information beyond the decision error threshold z^0; see part H of the appendix below), which is the desired result for part (b) of Theorem 2.

To prove part (c), recall that for all $a\in A$, ρ_a^{XB} necessarily drops to one as ρ_a^B drops to one (where the latter happens as $z\to\infty$). Thus, for all $a\in A$, ρ_a^{XB} must eventually reach a maximum for some finite $\hat{z}>z^0$. This result combined with the result of section B of the appendix (that each ρ_a^{XB} has a single peaked shape that strictly falls once the peak is reached) together imply that the joint reliability of *all* actions is falling for $z>\hat{z}$. Hence, the marginal benefit from using more information is negative for $z>\hat{z}$.

B. Single peakedness of ρ_a^{XB}

To establish conditions implying that joint reliability is single peaked, let a be an arbitrarily selected action from A (so that γ of part A above now equals ρ_a^{XB}) and recall that the sign of $\dot{\gamma}$ is determined by the sign of the expression in (A.1). Rearranging this implies $\dot{\gamma}<0$ if the following inequality holds,

$$\frac{A\dot{\rho}+\dot{A}(\rho-1)}{B\dot{\rho}+\dot{B}(\rho-1)}>\frac{A(\rho-1)+1}{B(\rho-1)+1}=\gamma. \tag{A.4}$$

Let z_a^* denote the value of z where γ reaches its *first* local maximum (which must exist since part (b) of Theorem 1 implies γ rises above one as z exceeds z^0, yet γ eventually drops back to one as $z\to\infty$); that is, $\dot{\gamma}(z)>0$ for all $z<z_a^*$ and $\dot{\gamma}(z_a^*)=0$. Now suppose the left side of (A.4) is rising at z_a^*, so that the inequality of (A.4) will start being satisfied immediately after z increases beyond z_a^* (since this implies the right side of (A.4), γ, will also immediately start dropping). In the same way, if the left side of (A.4) continues to rise then its right side, γ, will continue falling; so that the left side exceeds the right side by an ever increasing margin. Consequently, a sufficient condition for γ to strictly fall after it *first* stops rising ($\dot{\gamma}<0$ for all $z>z_a^*$) is that the left side of (A.4) have a strictly positive derivative for all $z\geqq z^0$.

To see what this entails, differentiate the *left* side of A.4 (hereafter denoted LA4), where its sign is determined by the sign of the numerator of the resulting ratio expression. Thus, the numerator must be positive for LA4's derivative to be positive. By cancelling and rearranging terms, we obtain the following inequality for the numerator of LA4's derivative.

$$(2\dot{\rho}^2-(\rho-1)\ddot{\rho})(\dot{A}B-A\dot{B})+\dot{\rho}(\rho-1)(\ddot{A}B-A\ddot{B})+(\rho-1)^2(\ddot{A}\dot{B}-\dot{A}\ddot{B})>0. \tag{A.5}$$

First consider the second multiple of (A.5), $\dot{\rho}(\rho-1)(\ddot{A}B-A\ddot{B})$. If A and B rise and drop at decellerating rates to their respective limits of 1 and 0 (meaning $\ddot{A}<0$ and $\ddot{B}>0$) then $\ddot{A}B-A\ddot{B}<0$ must hold. Thus let us require,

$$\ddot{A}<0 \quad \text{and} \quad \ddot{B}>0 \quad \text{for all} \quad z\geqq z^0. \tag{A.6}$$

Also, since $\dot{\rho}<0$ and $\rho>1$ for all $z>z^0$, and since $\rho(z^0)=\infty$, then ρ must be strictly falling from above to its lower bound of one (so that $\rho>1$ for all $z\geqq z^0$). Consequently, $\dot{\rho}(\rho-1)<0$ for all $z\geqq z^0$. By combining the above results we have $\dot{\rho}(\rho-1)(\ddot{A}B-A\ddot{B})>0$.

In similar fashion consider the third multiple of (A.5) for whom a nonnegative sign requires $\ddot{A}\dot{B}-\dot{A}\ddot{B}\geqq 0$ for $z\geqq z^0$. This will hold if and only if,

$$\left|\frac{\ddot{A}}{\dot{A}}\right|\geqq\left|\frac{\ddot{B}}{\dot{B}}\right| \quad \text{for} \quad z\geqq z^0. \tag{A.7}$$

This means that the absolute percentage rate of decrease in B toward 0 must not exceed the absolute percentage rate of increase in A toward 1.

Finally, consider the first term in (A.5). The expression $\dot{A}B-A\dot{B}$ is strictly positive since $A>0$, $B>0$. $\dot{A}>0$. $\dot{B}<0$ for all $z\geqq\bar{z}$. In addition, since $\dot{\rho}<0$ for $z>z^0$, the other expression $2\dot{\rho}^2-(\rho-1)\ddot{\rho}$ will be nonnegative if and only if

$$\frac{-2\dot{\rho}}{(\rho-1)}=\left|\frac{2\dot{\rho}}{\rho-1}\right|\geqq\frac{\ddot{\rho}}{-\dot{\rho}} \quad \text{for all} \quad z>z^0. \tag{A.8}$$

This means that minus the percentage change in the *derivative* of $(\rho-1)$ does not exceed twice the absolute percentage change in $(\rho-1)$. By combining conditions (A.6), (A.7), and (A.8) we then have sufficient conditions for LA4 to be strictly positive for all $z>z^0$ (thereby also representing sufficient conditions for γ to be single peaked for any $a\in A$).

C. *Effects of more competent decisions*

Next consider the effects of agents becoming more competent (or self-confident; see footnote 7) decision makers by using more costly information processing equipment and procedures. As before let action a be an arbitrary element of A. Since ρ_a^X rises strictly with z over $[\bar{z},\infty)$ there exists an inverse function, denoted ρ_a^{-1}, such that $\rho_a^X=\rho_a^X(z)$ implies $z=\rho_a^{-1}(\rho_a^X)$ for all $\rho_a^X\in[1,\infty)$. Thus define $\bar{\rho}_a^{XB}(\rho_a^X)\equiv\rho_a^{XB}(\rho_a^{-1}(\rho_a^X))\equiv\rho_a^{XB}(z)$. The properties of ρ_a^{XB} discussed in section 2 imply it equals ρ_a^X as ρ_a^X rises to the threshold at which decision errors begin for action a, denoted $\rho_a^0=\rho_a^X(z_a^0)$, such that $\dot{\rho}_a^B(z)<0$ for $z>z_a^0\geqq z^0$ (where z^0 of Theorem 2 equals the *infimum* of all z_a^0 for all $a\in A$). Hence, the properties of ρ_a^B imply it starts falling below ρ_a^X as z increases beyond z_a^0. The difference, $\rho_a^X-\bar{\rho}_a^{XB}(\rho_a^X)$, identically equals zero for $\rho_a^X\in[1,\rho_a^0]$, after which this difference grows steadily larger as ρ_a^X rises beyond ρ_a^0.

Now suppose that higher C_d both delays the error threshold ρ_a^0 and reduces the rate of decision errors beyond ρ_a^0, thereby slowing the drop in $\bar{\rho}_a^{XB}$ below ρ_a^X as the latter rises beyond ρ_a^0. Thus, ρ_a^0 is a strictly increasing function of C_d, denoted $\rho_a^0(C_d)$. To capture the second effect for ρ_a^X beyond ρ_a^0, let $h_a=\rho_a^X-\rho_a^0(C_d)$ and write $\bar{\rho}_a^{XB}$ as a function of both ρ_a^X and C_d, denoted $\bar{\rho}_a^{XB}(\rho_a^X,C_d)\equiv\bar{\rho}_a^{XB}(h_a+\rho_a^0(C_d),C_d)\equiv\beta_a^{XB}(h_a,C_d)$.

The function β_a^{XB} measures agents' joint reliability for each difference h_a in ρ_a^X *beyond* $\rho_a^0(C_d)$, and each level of decision costs C_d (which reduces the rate of drop of β_a^{XB} below ρ_a^X for each $h_a>0$). The latter effect means the cross-partial derivative of the difference $\rho_a^X-\beta_a^{XB}$ with respect to C_d and h_a is negative for $h_a>0$; that is, $\partial^2(\rho_a^X-\beta_a^{XB})/\partial C_d\partial h_a<0$ for $h_a>0$. This derivative reduces to $-\partial^2\beta_a^{XB}/\partial C_d\partial h_a$, which in turn implies,

$$\frac{\partial^2\beta_a^{XB}}{\partial C_d\partial h_a}>0 \quad \text{for all} \quad h_a>0 \quad \text{and} \quad a\in A. \tag{A.9}$$

The proof of single peakedness in part B also implies that for each given C_d, β_a^{XB} reaches a maximum for some finite $h_a^*(C_d)>0$. The objective is to show that positive marginal benefits from using information to select actions in A will continue *further* into the ID-zone as they become more competent at responding to information due to higher decision costs C_d. This requires showing that dh_a^*/dC_d is *positive* for all $a\in A$. To do so, consider the first and second order maximum conditions for h_a^*, which imply

$$\frac{\partial\beta_a^{XB}(h_a^*(C_d),C_d)}{\partial h_a}\equiv 0 \quad \text{for all} \quad C_d \quad \text{and} \quad a\in A \quad \text{and} \tag{A.10}$$

$$\frac{\partial^2\beta_a^{XB}}{\partial h_a^2}<0 \quad \text{at} \quad h_a^*(C_d) \quad \text{for all} \quad C_d \quad \text{and} \quad a\in A. \tag{A.11}$$

Thus, solving (A.10) implicitly for dh_a^*/dC_d and recalling (A.9) and (A.11) implies $dh_a^*/dC_d=(-\partial^2\beta_a^{XB}/\partial C_d\partial h_a)/(\partial^2\beta_a^{XB}/\partial h_a^2)>0$ for all $a\in A$, which is the desired result.

D. Proof of Theorem 1 for ρ_a^{XB}

If $s\in S_a^*$ occurs, agents can end up choosing action a either by messages X_a^* 'correctly' signaling S_a^* and agents 'correctly' responding to X_a^* by selecting $B(x)$ such that $a\in B(x)$; or by messages $X-X_a^*$ occurring instead and agents still responding to $X-X_a^*$ by choosing $B(x)$ such that $a\in B(x)$. Thus, $p(a\in B(x)\,|\,S_a^*)=r_a^{XB}=r_a^Xr_a^B+(1-r_a^X)w_a^B$. Similarly, if $s\in S-S_a^*$ occurs agents may also select action a if messages X_a^* still arise and agents respond to them by selecting $B(x)$ such that $a\in B(x)$; or if messages $X-X_a^*$ arise and agents still respond to them by choosing $B(x)$ such that $a\in B(x)$. Thus $p(a\in B(x)\,|\,S-S_a^*)=w_a^{XB}=w_a^Xr_a^B+(1-w_a^X)w_a^B$. Next divide the formula for w_a^{XB} into the preceding formula for r_a^{XB}, and rearrange terms to obtain,

$$\frac{r_a^{XB}}{w_a^{XB}}=\frac{r_a^X(r_a^B-w_a^B)+w_a^B}{w_a^X(r_a^B-w_a^B)+w_a^B}. \tag{A.12}$$

Formula (1) of the text follows immediate by dividing both the numerator and denominator of (A.12) by w_a^B.

E. Notation changes to compare with information theory

In order to reduce the complexity of the remaining analysis, certain previous definitions and notation are changed. Doing so will also make more explicit the connection to basic concepts of information theory and cybernetics.

For each $s \in S$, let A_s denote the set of all actions in A with maximal utility if selected when s occurs. That is, $A_s = \{a' \in A \mid U(a',s) \geqq U(a,s)$ for all $a \in A\}$, where $U(\cdot)$ denotes the agent's utility function. Next partition S into subsets whose elements produce the *same* A_s sets, and assume (for analytic convenience) that the resulting partition is countable (so that the subsets of the partition can be indexed with $i=(1,2,\ldots)$ denoted S_i). We then have $S_i = \{s, s' \in S \mid A_s = A_{s'}\}$, $S_i \cap S_j = \phi$ for all $i \neq j$, and the union over S_i equals S. Note also that since A_s is the same for all $s \in S_i$, then it can be correspondingly denoted A_i for each i. Finally, let $X_i \subset X$ denote the subset of messages for which A_i contains all those actions that maximize expected utility given $x \in X_i$ (that is, $X_i = \{x \in X \mid B^*(x) = A_i\}$).

The sets S_i, X_i, and A_i are now interpreted as individual 'events' corresponding to particular types of: states occurring, messages being observed, and actions being selected (i.e., state-events, information-events, and decision-events). We can then apply the concepts of information theory, where an agent's decisions in response to observed messages are interpreted as the outputs of a communication channel. Information events X_i represent *inputs* to the channel and decision events A_j represent *outputs* of the channel (where a 'j' subscript is used to distinguish outputs from inputs, which are denoted with a subscript 'i').

Such a channel between information and decision events X_i and A_j will ereafter be called a 'decision channel'. A perfect decision channel (one that lways outputs the optimal decision event A_i for each message input event X_i) would perfectly correlate each input X_i with the *same* indexed output A_i. That is, the following *degenerate* conditional output probabilities would result.

$$p(A_i \mid X_i) = 1 \quad \text{for all} \quad i, \tag{A.13a}$$

$$p(A_j \mid X_i) = 0 \quad \text{whenever} \quad j \neq i. \tag{A.13b}$$

We can then define the probabilities of 'rightly' instead of 'wrongly' outputing decisions from received input messages (analogous to the chances of rightly or wrongly responding to observed messages in the main text). Let

$$r_j^B = p(A_j \mid Y \subset X_j), \tag{A.13c}$$

$$w_j^B = p\left(A_j \mid Y \subset \sum_{i \neq j} X_i\right) = p(A_j \mid Y \subset X - X_j). \tag{A.13d}$$

A *finite* capacity decision channel can only imperfectly correlate inputs with their corresponding optimal output responses; so that $r_j^B < 1$ and $w_j^B > 0$.

We can now think of reliability concepts of the main text in terms of Shannan's negative entropy measure of a channel's capacity to transmit information about which inputs have been received. The amount of information or uncertainty about which inputs may be potentially received equals,

$$H(X) = -\sum_i p(X_i) \ln p(X_i). \tag{A.14}$$

The amount of uncertainty about which input X_i was received, given the decision channel outputted A_j (see Shannon and Weaver (1963, hereafter denoted by SW; page 52)], equals

$$H_A(X) = -\sum_i \sum_j p(X_i, A_j) \ln p(X_i \mid A_j). \tag{A.15}$$

By using Bayes rule to reverse the conditional probabilities $p(X_i \mid A_j)$ in (A.15), it is easily shown that an imperfect agent with finite reliability $\rho_j^B = r_j^B / w_j^B < \infty$ implies $H_A(X)$ is *positive* (meaning its output decisions imperfectly reveal whether input messages have been received for which outputs are optimal choices). That is,

$$\rho_j^B < \infty \quad \text{for some} \quad A_j \quad \text{implies} \quad H_A(X) > 0. \tag{A.16a}$$

On the other hand, a perfect agent with infinite reliability implies $H_A(X) = 0$; that is,

$$\rho_j^B = \infty \quad \text{for all} \quad A_j \quad \text{implies} \quad H_A(X) = 0. \tag{A.16b}$$

The capacity, denoted C, of an imperfect (decision) channel to transmit information is (see SW, page 70),

$$C = \max(H(X) - H_A(X)). \tag{A.17a}$$

Finally, let the variable z now index changes in the amount of information agents might try to interpret as input. A simple way to accomplish this is to set z equal to $H(X)$. That is, let

$$z = H(X), \quad \text{where} \quad \bar{z} = \min z = 0. \tag{A.17b}$$

F. Finite channel capacity implies a finite decision error threshold exists

One of the fundamental theorems of information theory (see SW, Theorem 11 on page 71) is that if the amount of information received as input does not exceed an imperfect channel's finite capacity C (so that $H(X)\leqq C$), then there always exists a way to transmit received messages so that $H_A(X)$ comes arbitrarily close to zero (see also fig. 9 of SW, page 71).[10] Since $H_A(X)=0$ corresponds to $\rho_j^B=\infty$ for all A_j, and since $z=H(X)$ by (A.17b), then the theorem implies there exists an interval $[\bar{z},z^0]=(0,C)$ such that ρ_j^B can be made arbitrarily large for all A_j. Thus, the interval $(0,C)$ can be interpreted as the perfect decision or PD-zone discussed in the main text. As z rises beyond $z^0=C$, Theorem 11 of SW implies $H_A(X)>0$, which in turn implies $\rho_j^B<\infty$ for some A_j. Hence, the interval $(z^0,\infty)=(C,\infty)$ corresponds to the imperfect decision on ID-zone discussed in the main text.

G. Measuring information complexity with $H(X)$

$H(X)$ is a measure of the number and likelihood of potential messages from an information source (see SW, pages 36–42). $H(X)$ thus also measures the complexity of an information source. As $H(X)$ increases, agents must try and interpret ever increasing numbers of perhaps subtly differentiated messages, most or all of which arise with negligible probability. We can also define,

$$H(S)=-\sum_k p(S_k)\ln p(S_k), \tag{A.18a}$$

as a measure of the complexity of the environment encountered by agents. Larger $H(S)$ corresponds to larger numbers of state-events S_k for which different action events A_k must occur in order to always select a best action for all potential environmental states (recall that S_k contains all those states for which actions from A_k are optimal). As $H(S)$ rises, ever increasing number of events S_k will potentially occur, which are distinguishable from each other only according to more complex and subtle factors.

It is easy to show in general that $H(X)+H_X(S)=H(S)+H_S(X)$ (see SW, page 52), which implies

$$H(X)\geqq H(S)+H_S(X), \tag{A.18b}$$

[10]The original interpretive discussion by Shannan focused on an imperfect communication channel rather than an imperfect decision channel as here discussed. However, the method of proof of Theorem 11 of SW (1963) is independent of any physical property about the actual process of transmission through a channel, no matter how the latter might be interpreted (it is a nonconstructive, 'existence' proof derived only using formal probability relationships independent of any physical process that might give rise to them). It can thus be applied to widely differing cases (such as electronic signals, human language, the genetic code embodied in DNA molecules, and so on), including both sending messages to an agent as well as agents responding to received messages.

since $H_X(S) \geqq 0$ always holds. Similar to the above discussion for $H_A(X)$ in part F, it can be shown that $\rho_j^X \to \infty$ for all A_j implies $H_S(X) \to 0$. Hence, from (A.18b) we have.

$$\rho_j^X \to \infty \quad \text{for all} \quad A_j \quad \text{implies} \quad H(X) \to H^* \geqq H(S). \tag{A.19}$$

Implication (A.19) means that in order to become perfectly reliable, *an information source must thereby also become at least as complex and subtly differentiated as the environmental events S_k which it attempts to predict.* Thus, a complex environment may place severe demands on agents' ability to interpret information that reliably (and in the limit perfectly) predicts all the environmental events S_k that are relevant to determining their best choices.

This result also relates to the 'law of requisite variety' (see Ashby 1963, pages 206–218). In the present context, it means that an information source must produce messages with at least as much 'variety' as the events S_k in order for agents potentially to be able to reduce the *difference* between their actually selected and optimal actions down to zero; thereby enabling agents to vary their decisions so as to continually maintain this difference equal to zero (that is, to vary their decisions so as to continually make optimal responses to varying environmental conditions).

H. Joint reliability and marginal benefits to using information

The relationship between joint reliability ratios and the marginal benefit from using more reliable information is now discussed using the above information theory concepts. Doing so will simplify the formal analysis, while still focusing on the essential relationships involved.

To do so, let $X - X_i = \sum_{k \neq i} X_k$ (meaning message events for which other decision events besides A_i are optimal given those messages have occurred) and $S - S_i = \sum_{j \neq i} S_j$ (meaning state events for which other action events besides A_i are optimal if the events $S - S_i$ were perfectly known to agents). Then denote the conditional information and decision probabilities of the main text as (also recalling they depend on the information index z, such as $z = H(X)$ discussed above in part G).

$$r_i^B(z) = p(A_i \mid Y \subset X_i) \quad \text{and} \quad w_i^B(z) = p(A_i \mid Y \subset X - X_i), \tag{A.20a}$$

$$r_i^X(a) = p(X_i \mid Y \subset S_i) \quad \text{and} \quad w_i^X(z) = p(A_i \mid Y \subset S - S_i), \tag{A.20b}$$

$$r_i^{XB}(z) = p(A_i \mid Y \subset S_i) \quad \text{and} \quad w_i^{XB}(z) = p(A_i \mid Y \subset S - S_i), \tag{A.20c}$$

$$\rho_i^{XB} = r_i^{XB} / w_i^{XB}, \rho_i^X = r_i^X / w_i^X, \rho_i^B = r_i^B / w_i^B. \tag{A.20d}$$

By similar reasoning to that of part D above, we can also express the joint probabilities $r_i^{XB}(z)$ and $w_i^{XB}(z)$ as,

$$r_i^{XB}(z)=r_i^B(z)r_i^X(z)+w_i^B(z)(1-r_i^X(z)), \tag{A.21a}$$

$$w_i^{XB}(z)=r_i^B(z)w_i^X(z)+w_i^B(z)(1-w_i^X(z)). \tag{A.21b}$$

Hence the ratio (A.21a)/(A.21b) can be transformed into an analogous formula to eq. (1) of the main text.

Let $U(A_i,S_j)$ denote the utility to an agent from selecting an action within A_i when event S_j occurs. Recalling that A_i contains all the best actions for event S_i, we then have

$$U(A_i,S_i)>U(A_i,S_j) \quad \text{for all} \quad i \quad \text{whenever} \quad j\neq i. \tag{A.22}$$

Next define the expected utility[11] if an action from $A-A_i$ is selected when event S_i occurs,

$$EU(A-A_i\,|\,S_i)=\sum_{j\neq i}\frac{p(A_j\,|\,S_i)}{p(A-A_i\,|\,S_i)}\cdot U(A_j\,|\,S_i), \qquad \text{where} \tag{A.23a}$$

$$p(A-A_i\,|\,S_i)=\sum_{j\neq i}p(A_j\,|\,S_i). \tag{A.23b}$$

Since $EU(A-A_i\,|\,S_i)$ equals a convex combination of the set of utilities $U(A_j,S_i)$ for $j\neq i$, then it always lies between the maximum and minimum values of these utilities. Thus, by (A.22) we have,

$$U(A_i,S_i)>EU(S-S_i\,|\,A_i) \quad \text{for all} \quad i. \tag{A.24}$$

Next let $EU(A\,|\,X)$ represent the expected utility from selecting actions within A in response to observed messages in X. Standard analysis then implies

$$EU(A\,|\,X)=\sum_i\sum_j p(A_i,S_j)U(A_j,S_i)=\sum_i p(S_i)\left\{\sum_j p(A_j\,|\,S_i)U(A_j,S_i)\right\}$$

[11] The analysis developed here (and in the main text) does not depend on assuming expected utility, but can be generalized to allow for recent 'non-expected utility theories' of Machina, Chew, Fishburn, and others [see Machina (1983)]. This generalization is developed in Heiner (1985a).

$$=\sum_i p(S_i)\left\{p(A_i\,|\,S_i)U(A_i,S_i)+\sum_{j\neq i} p(A_j\,|\,S_i)U(A_j\,|\,S_i)\right\}$$

$$=\sum_i p(S_i)\left\{p(A_i\,|\,S_i)U(A_i,S_i)+p(A-A_i\,|\,S_i)EU(A-A_i\,|\,S_i)\right\}.$$

Since $1-p(A_i\,|\,S_i)=p(A-A_i\,|\,S_i)=\sum_{j\neq i} p(A_j\,|\,S_i)$, and since $S_i\subset S-S_j$ then definition (A.20c) implies

$$EU(A\,|\,X)=\sum_i p(S_i)\left\{r_i^{XB}(z)U(A_i,S_i)+\sum_{j\neq i} w_j^{XB}(z)EU(A-A_i\,|\,S_i)\right\}, \quad \text{(A.25a)}$$

where $\sum_{j\neq i} w_j^{XB}(z)=1-r_i^{XB}(z).$ (A.25b)

Each bracketed term { } of (A.25a) is a convex combination of $U(A_i,S_i)$ and $EU(A-A_i\,|\,S_i)$; where the former always exceeds the latter by (A.24). Thus, if the relative probability weights of all mistaken decisions shift toward the joint probabilities $r_i^{XB}(z)$ for all A_i (corresponding to $\rho_i^{XB}(z)$ rising for all A_i), then all the bracketed expressions { } of (A.25a) will rise.[12] Hence, for any given set of probabilities of different states arising, $p(S_i)$, $EU(AX)$ will rise if joint reliability $\rho_i^{XB}(z)$ increases for all i. Similar reasoning also implies that if joint reliability falls for all i, then $EU(A\,|\,X)$ must eventually also fall (that is, joint reliability simultaneously rising or falling for all i will produce positive or negative 'marginal benefits' from raising the index z toward using more reliable information).

[12]One can also separate $EU(AX)$ into another decomposition involving the 'posterior' expected utilities conditional on the occurrence of particular decision events A_i, denoted $EU(S\,|\,A_i)=\sum_j p(S_j\,|\,A_i)U(A_i,S_j)$. It can also be shown that each $EU(S\,|\,A_i)$ is a convex combination involving r_i^{XB} and w_i^{XB} such that raising ρ_i^{XB} will increase $EU(S\,|\,A_i)$. Thus, if joint reliability rises simultaneously for all i, then the posterior expected utilities conditional on decision events A_i will thereby also simultaneously rise for all i.

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